THE FIRST-ORDER DELTA-SIGMA MODULATOR

MOD 1

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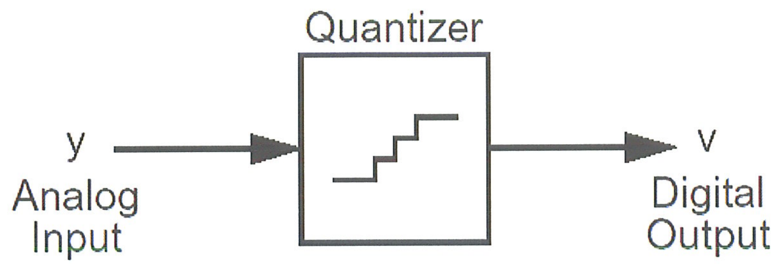
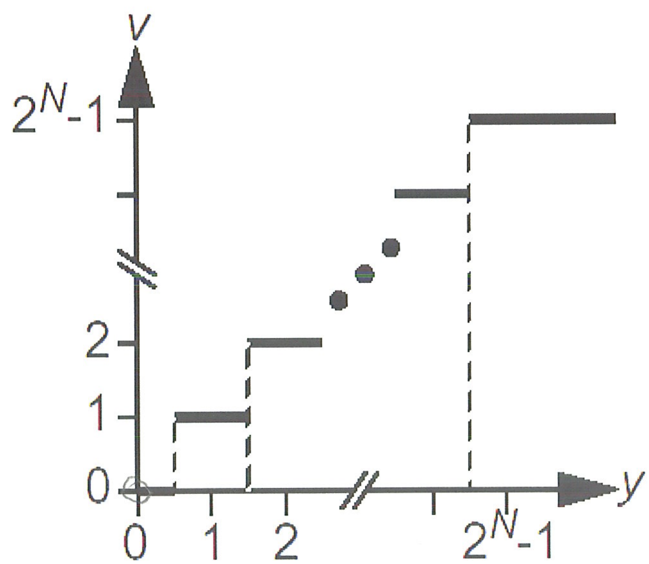
Outline

- Quantizers and quantization noise
- Binary quantization
- MOD1 as an ADC
- MOD1 as a DAC
- MOD1 linear model
- Simulation of MOD1
- MOD1 under DC excitation
- The effects of finite op-amp gain
- Decimation filters for MOD1

Quantizers and Quantization "Noise" (1)

- Unipolar N-bit quantizer:

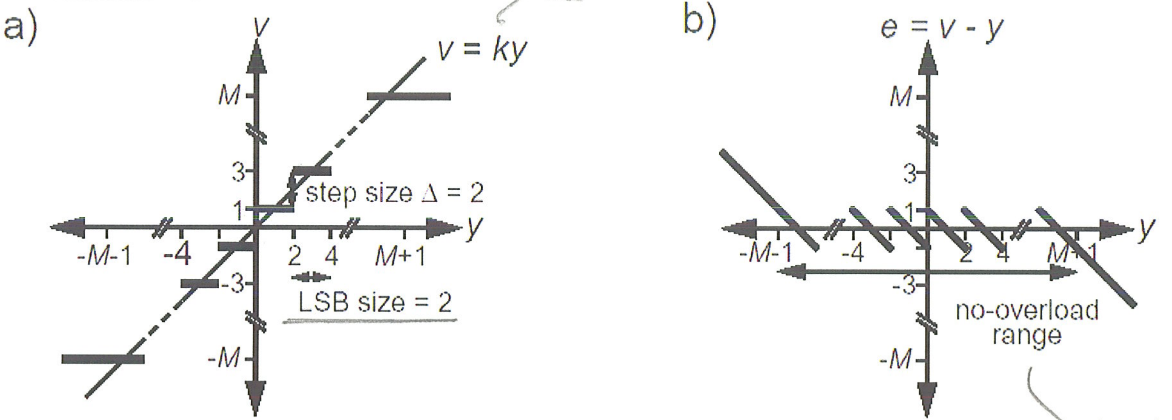




Midtread

Quantizers and Quantization "Noise" (2)

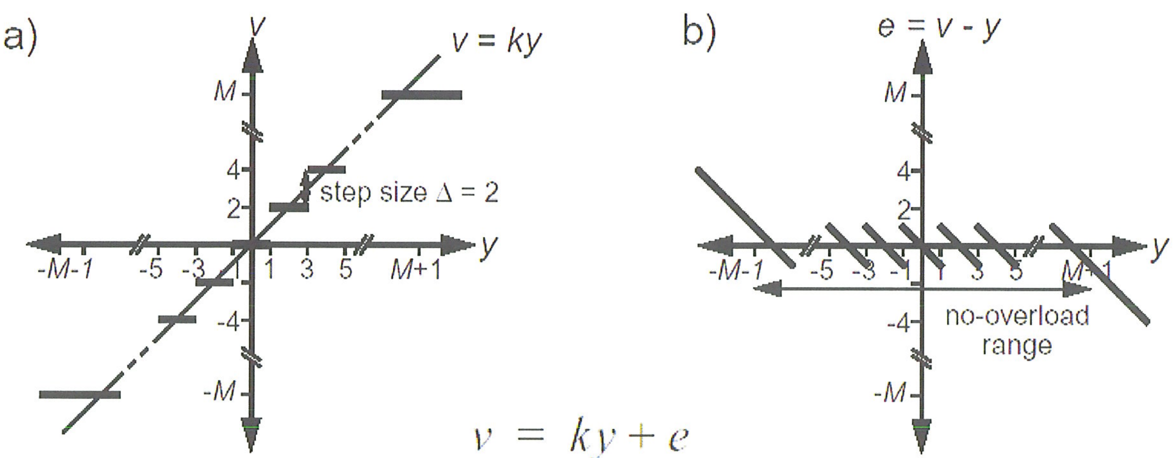
- M-step mid-rise quantizer: 1



$-(M+1) < y < M+1$

$|e| \leq 1$

- M-step mid-tread quantizer:

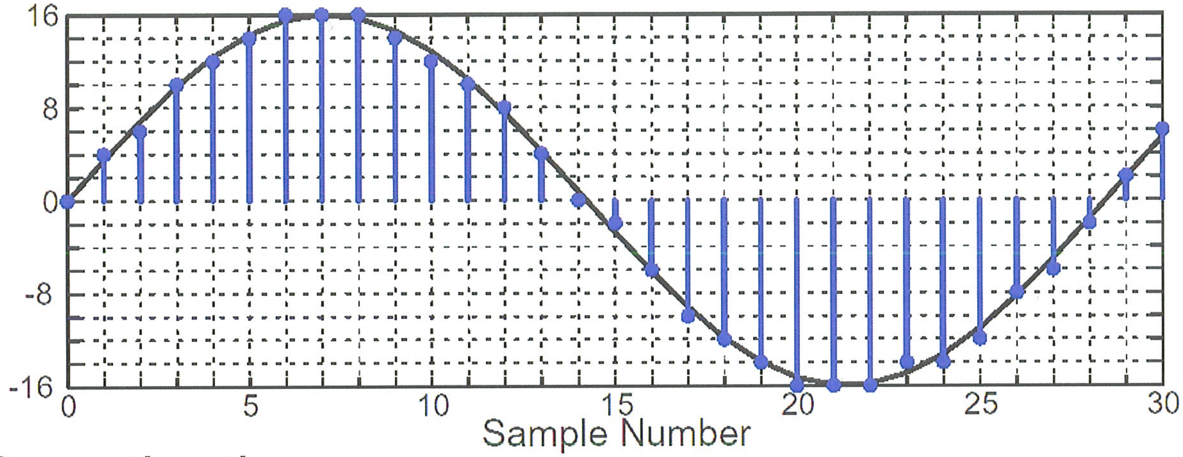


$v = ky + e$

Quantizers and Quantization "Noise" (3)

- Sampled signal:

y, v

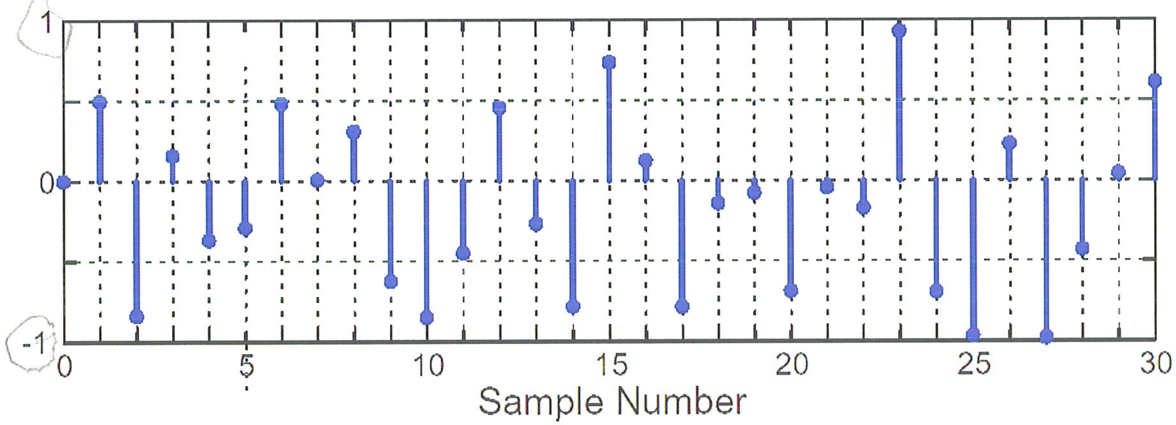


16-step quantization used

RMS of noise
$\Delta/\sqrt{12}$

- Quantization error:

e

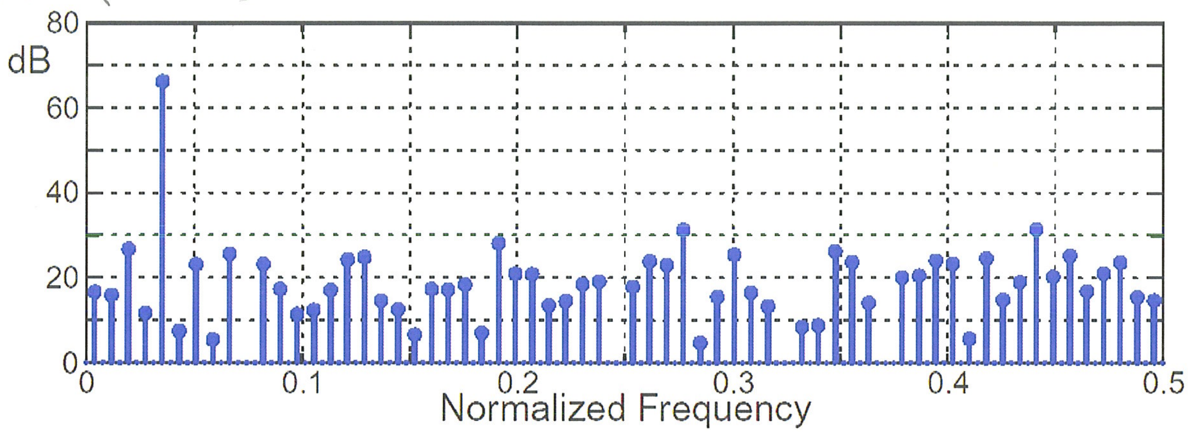


f/f_s irrational!

large input

Quantizers and Quantization Noise (4)

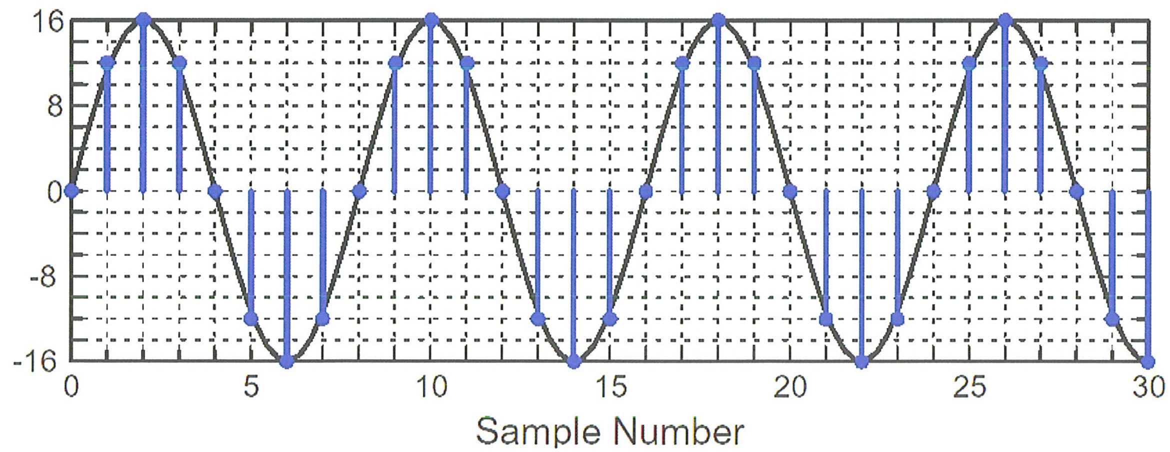
- FFT: of e(n)



~ white

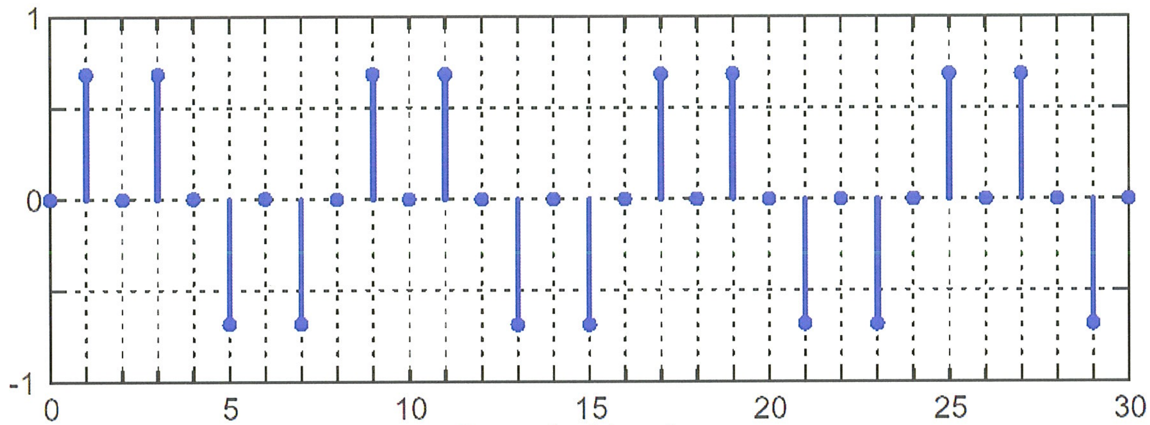
rational

- Sampled signal ($f = f_s/8$):

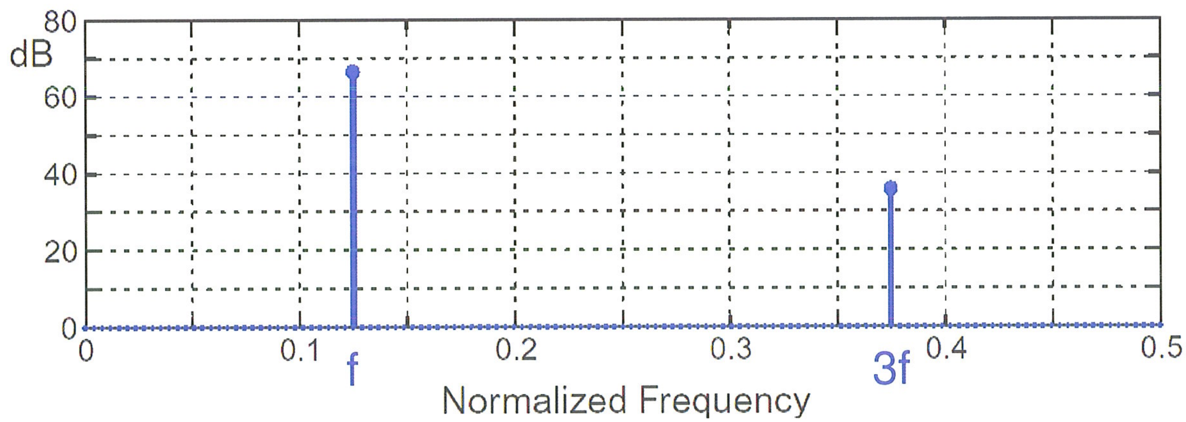


Binary Quantization (1)

- Quantization error:

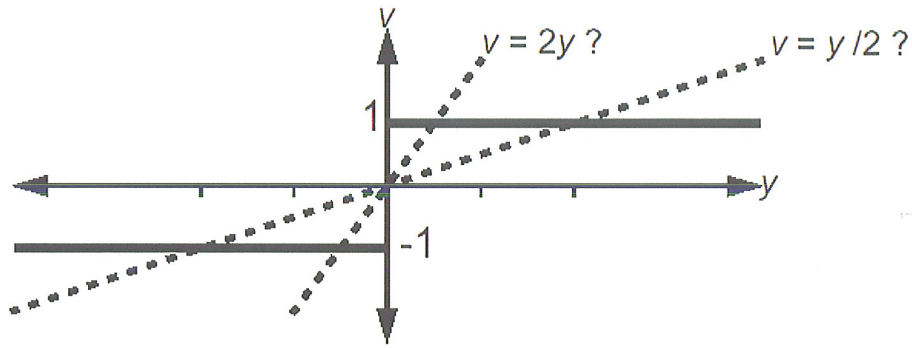


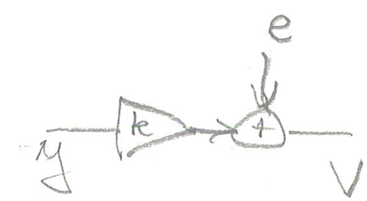
- FFT:



Binary Quantization (2)

- Modeling the gain:





- Minimize mean square error of *e*: $$\sigma_e^2 = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} e(n)^2$$

"Inner product"
"Scalar — " — "

$$\langle a, b\rangle \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} a(n)b(n) = E[ab]$$

$$\begin{aligned}\sigma_e^2 &= \langle e, e\rangle \\ &= \langle v - ky, v - ky\rangle \\ &= \langle v, v\rangle - 2k\langle v, y\rangle + k^2\langle y, y\rangle.\end{aligned}$$

opt.: $k = \dfrac{\langle v, y\rangle}{\langle y, y\rangle} = \dfrac{E[|y|]}{E[y^2]}$

Decorrelation: $\langle y, e\rangle \overset{!}{=} 0$

$\langle y, e\rangle = \langle y, v - ky\rangle = \langle y, v\rangle - k\langle y, y\rangle$

same result

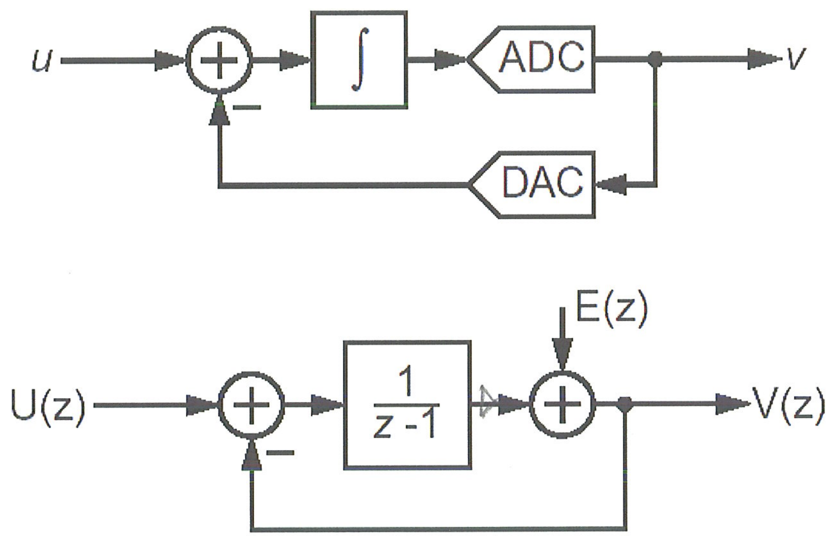
Check it for sinewave!

MOD1 as an ADC (1)

- Linear modeling:

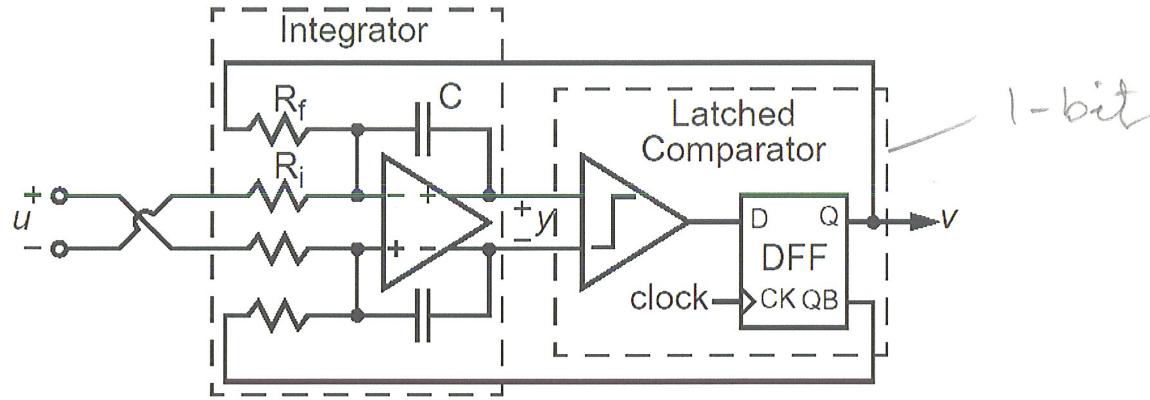
(a)

(b)

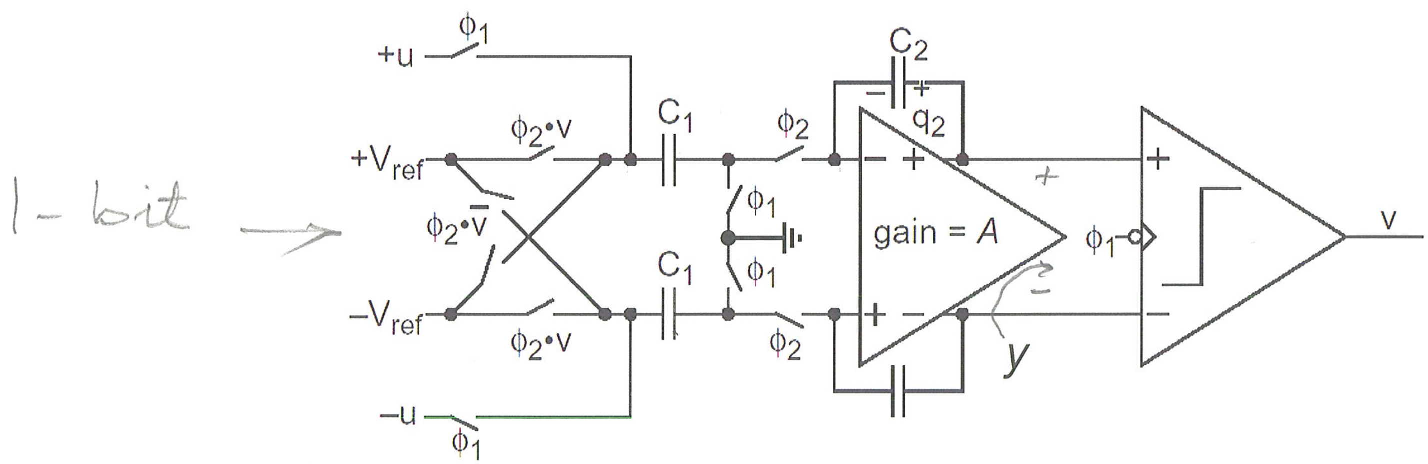


MOD1 as an ADC (2)

- Continuous-time implementation:

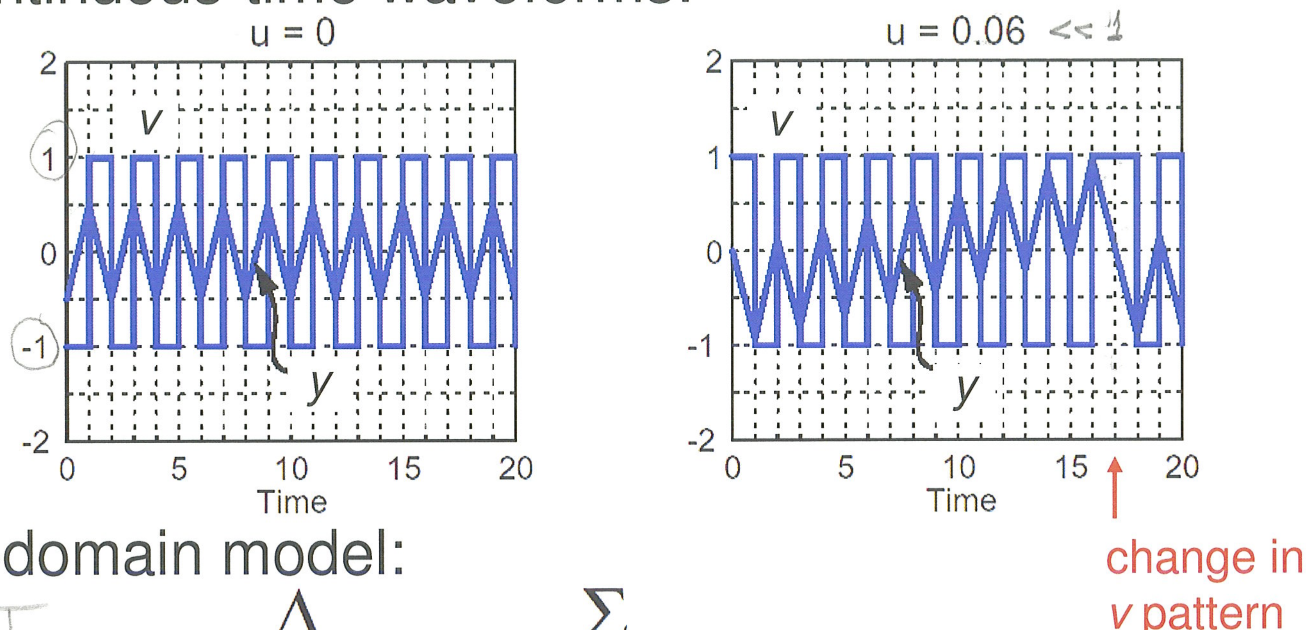


- Discrete-time switched-capacitor implementation:



MOD1 as an ADC (3)

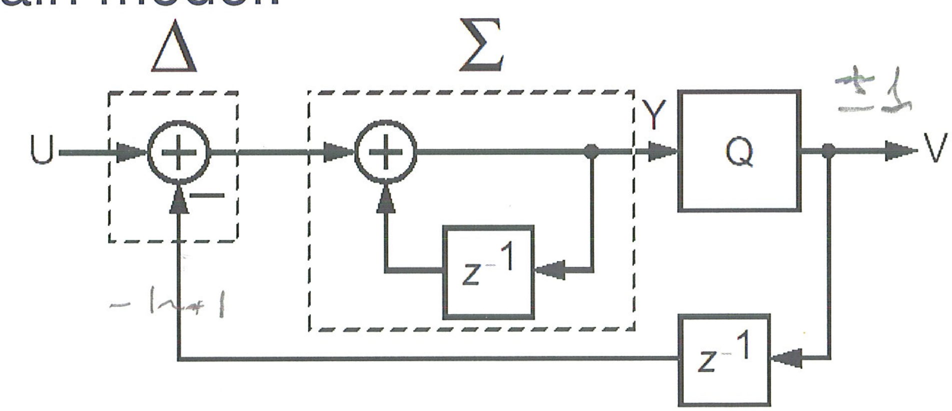
- Continuous-time waveforms:



V(0) → 1

- Z-domain model:

DT

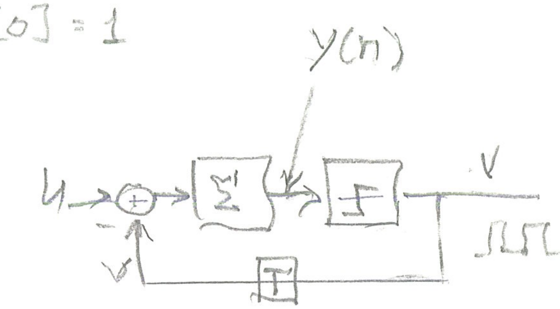


MOD1 as an ADC (4)

- Stable operation:

Initial condition:
$y(-1) = 0$

$$v(n) = \mathrm{sgn}[y(n)].$$ $\mathrm{sgn}[0] = 1$



$$y(n) = y(n-1) + u(n) - v(n-1)$$

$$y(N) - y(0) = \sum_{n=0}^{N} [u(n) - v(n-1)].$$

$V_{ref} = 1V$

If $y(n)$ is bounded,

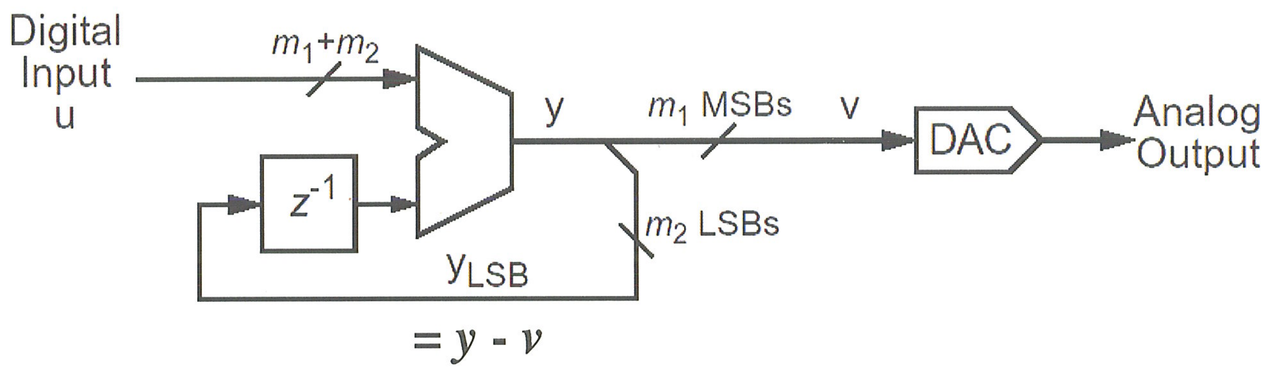
$$\lim_{N \to \infty} \frac{y(N) - y(0)}{N} = 0$$

$$\overline{u} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} u(n) = \overline{v} \quad !$$

Perfectly accurate for $N \to \infty$, for $H(0) \to \infty$

MOD1 as a DAC

- Error feedback structure: → recycled error!



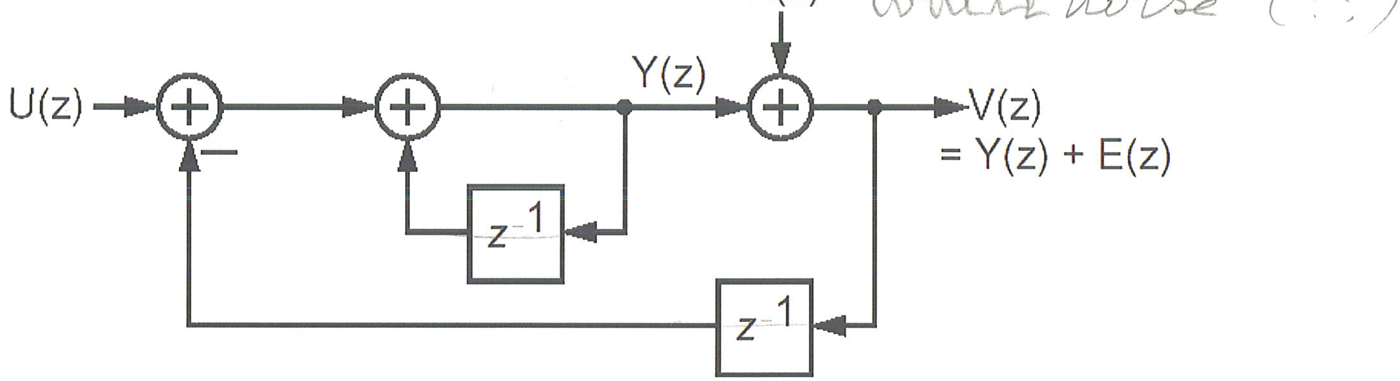
$$y(n) = u(n) + y_{LSB}(n-1)$$

$$y(n) = u(n) + y(n-1) - v(n-1)$$

Same as for ΔΣ loop → another option for DAC.
(For ADC, impractical!)

MOD1 Linear Model (1)

- Z-domain analysis:



$$Y(z) = z^{-1}Y(z) + U(z) - z^{-1}V(z)$$

$$\begin{aligned} V(z) &= Y(z) + E(z) = z^{-1}Y(z) + U(z) - z^{-1}V(z) + E(z) \\ &= U(z) + E(z) - z^{-1}(V(z) - Y(z)) \\ &= U(z) + E(z) - z^{-1}E(z) \\ &= U(z) + (1 - z^{-1})E(z). \end{aligned}$$

$$V(z) = STF(z)U(z) + NTF(z)E(z)$$

MOD1 Linear Model (2)

- Frequency-domain analysis: $|NTF(e^{j2\pi f})|^2 = [2\sin(\pi f)]^2 \simeq (2\pi f)^2$

$NTF = 1 - z^{-1}$

$z \to e^{j\omega T}$

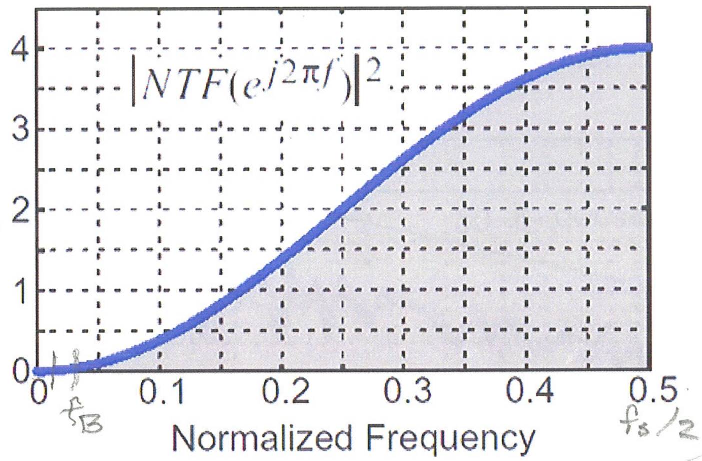
$f \ll f_s = 1$

$T = 1$

$0 - f_B$

$OSR = f_s/(2f_B) \gg 1$

$f_B = 1/(2 \cdot OSR)$



white PSD of $e(n)$

Mean square of q_o: $\sigma_{q_0}^2 \cong \int_0^{1/(2 \cdot OSR)} [2\pi f]^2 S_e(f) df = \dfrac{\pi^2}{9(OSR)^3}$ (for OSR >> 1)

(inband shaped quant. noise)

power

FS sine wave

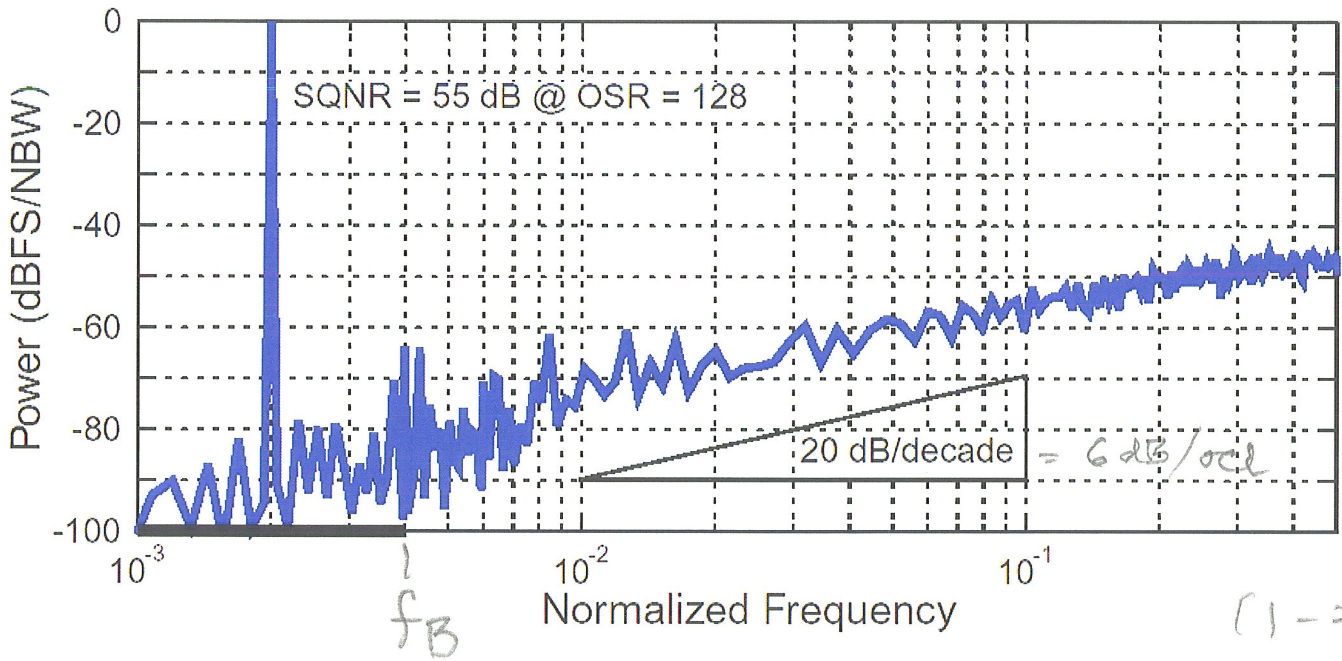
\# of levels in Q

$$\boxed{SQNR = \frac{\sigma_u^2}{\sigma_{q_0}^2} = \frac{9M^2(OSR)^3}{2\pi^2}}$$

Signal-to-quantization noise ratio

Simulation of MOD1 (1)

- Output spectrum for full-scale sine-wave input:



Plot $\int_0^f PSD\,df$

$(1-z^{-1})$

Looks ok, but SQNR 5 dB less than the formulaic.